Global Education of Science

Subject : Mathematics
Standard : 10
Total Mark : 100

MCQ and Subjective

Paper Set : 1
Date : 26-07-2024
Time : 0H:20M

Mathematics - Section A (MCQ)

(1) The distance from the origin of $A(a\cos\theta, a\sin\theta)$ is .... $(a \in R^+)$

(A) $a\cos\theta$ (B) $a\sin\theta$
(C) a (D) 1

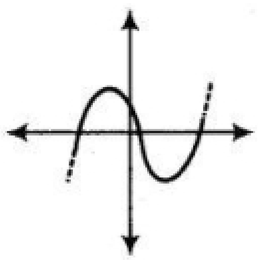
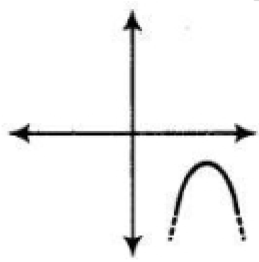
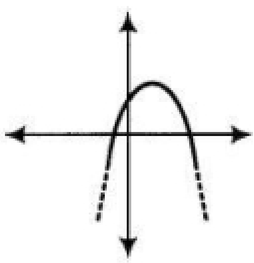
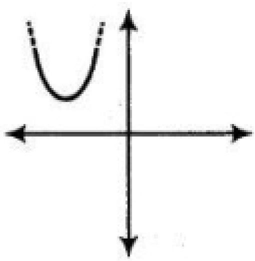
(2) The sum of the two−digit number and the number obtained by interchanging the digits is always divisible by ....

(A) 9 (B) 10
(C) 11 (D) 12

(3) Find the roots of the following quadratic equations by using the general formula for the roots, if they exist :
$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$; $(x \neq -1, -2, -4)$

(A) $-2(1+\sqrt{3})$ and $-2(1+\sqrt{3})$
(B) $2(1-\sqrt{3})$ and $2(1+\sqrt{3})$
(C) $2(1+\sqrt{3})$ and $2(1-\sqrt{3})$
(D) $-2(1-\sqrt{3})$ and $-2(1-\sqrt{3})$

(4) Which of the following is not the graph of a quadratic polynomial?

(A)

(B)

(C)

(D)


(5) $\Delta ABC \sim \Delta PQR$ for the correspondence $ABC \leftrightarrow PQR$. If $AB = 4, BC = 8, AC = 10$ and $PR = 15$, then the perimeter of $\Delta PQR = ...$

(A) 25 (B) 33
(C) 40 (D) 60

(6) Solve the following pairs of linear equations in two variables using graph : $2x + y = 7, \quad x - 2y = 6$

(A) $(-6, 3)$ (B) $(-3, 2)$
(C) $(4, -1)$ (D) $(7, 1)$

(7) Which of the following group truely match the data of Part I with the data of Part II?

Part I	Part II
1. In $\Delta ABC, AB = 3, BC = 4$ and $AC = 5$	a. in-radius 2
2. In $\Delta PQR, PQ = 5, QR = 12$ and $PR = 13$	b. in-radius 1
3. In $\Delta XYZ, XY = 8, YZ = 15$ and $XZ = 17$	c. in-radius 6
4. In Δ MNP, $MN = 2O, NP = 21$ and $MP = 29$	d. in-radius 3

(A) $(1-a), (2-b), (3-c), (4-d)$
(B) $(1-c), (2-d), (3-a), (4-b)$
(C) $(1-d), (2-c), (3-b), (4-a)$
(D) $(1-b), (2-a), (3-d), (4-c)$

(8) A circle touches the sides $\overline{AB}, \overline{BC}$ and $\overline{CA}$ of ΔABC at the points D, E, F respectively. If $AB = 13, BC = 12$ and $CA = 5$, then $AD = ...$

(A) 2 (B) 5
(C) 3 (D) 10

(9) In $\Delta PQR, m\angle Q = 90$ and $\overline{QM}$ is an altitude. If $PM = 8$ and $RM = 12$, then $QM = ...$

(A) $4\sqrt{6}$ (B) $8\sqrt{3}$
(C) 10 (D) 18

(10) .... is the value of the k, if one of the roots of the quadratic equation $x^2 + 6x + k = 0$ is 4.

(A) 20 (B) 40
(C) 8 (D) -40

(11) A ladder is leaning against a wall such that its upper end touches the wall at the height $3m$ and the ladder is inclined at an angle having measure 30 with the ground. Find the length of the ladder. (in m)

(A) 6 (B) 8
(C) 4 (D) 12

(12) Find the roots of the quadratic equations by using the quadratic formula
$x^2 + 2\sqrt{2}x - 6 = 0$

(A) $\sqrt{7}, -2\sqrt{3}$ (B) $\sqrt{5}, -2\sqrt{3}$
(C) $\sqrt{2}, -2\sqrt{3}$ (D) $\sqrt{2}, -3\sqrt{2}$

(13) In $\Delta ABC, A-M-B, A-N-C$ and $\overline{MN} \| \overline{BC}$. If $AM = x+3, AB = 2x, AN = x+5$ and $AC = 2x+3$, find the value of x.

(A) 3 (B) 5
(C) 9 (D) 1

(14) The perimeter of rhombus $ABCD$ is 68 . If $AC = 30$, find BD

(A) 16 (B) 30

(C) 35 (D) 45

(15) Solve the following pairs of linear equations in two variables using graph : $x+y=8,\ x-y=2$

(A) $(5,\ 3)$ (B) $(-5,\ 3)$

(C) $(5,\ -3)$ (D) $(-5,\ -3)$

(16) The degree of the polynomial $p(x)=x^2-x^3+x+1$ is ....

(A) -3 (B) 3

(C) 2 (D) 1

(17) Which of the following equations has two distinct real roots?

(A) $2x^2-3\sqrt{2}x+\frac{9}{4}=0$ (B) $x^2+3x+2\sqrt{2}=0$

(C) $x^2+x-5=0$ (D) $5x^2-3x+1=0$

(18) In $\Delta PQR, \overline{PS}$ is a median. If $PQ=9, PR=40$ and $PS=20.5$, find QR.

(A) 41 (B) 30

(C) 35 (D) 50

(19) Find the sum of last ten terms of the $AP: 8, 10, 12, \ldots, 126$.

(A) 1150 (B) 1160

(C) 1170 (D) 1180

(20) The product of the zeros of $p(x)=4x^2+12x+5$ is.........

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$

(C) $\frac{3}{4}$ (D) $\frac{4}{3}$

(21) Solve the following equations using the method of factorization : $x-\frac{1}{x}=\frac{45}{14} \quad (x\neq 0)$

(A) $-\frac{7}{2}$ and $-\frac{2}{7}$ (B) $\frac{7}{2}$ and $\frac{2}{7}$

(C) $\frac{7}{2}$ and $-\frac{2}{7}$ (D) $-\frac{7}{2}$ and $\frac{2}{7}$

(22) g.c.d. $(18,24)\times 1.c.m.(18,24)=\ldots\ldots\ldots\ldots$

(A) 144 (B) 72

(C) 432 (D) $6\times 18\times 24$

(23) $\Delta XYZ\sim\Delta DEF$ for the correspondence $XYZ\leftrightarrow EDF$. If $XY=3, YZ=4, ZX=6$ and $DF=12$, find the perimeter of ΔDEF.

(A) 50 (B) 45

(C) 39 (D) 60

(24) $(0,0), (3.1,0)$ and $(0,4.5)$ are the vertices of .... triangle.

(A) an equilateral (B) a right angled

(C) an isosceles (D) an acute angled

(25) Find the square root in the form of binomial surd : $6+\sqrt{35}$

(A) $\frac{\sqrt{6}+\sqrt{5}}{2}$ (B) $\frac{\sqrt{4}+\sqrt{10}}{3}$

(C) $\frac{\sqrt{14}+\sqrt{10}}{2}$ (D) $\frac{\sqrt{10}+\sqrt{14}}{2}$

(26) If 64 is the discriminant of $kx^2-4x-4=0$, then the value of k is ....

(A) -4 (B) 4

(C) 3 (D) -3

(27) In $\Delta ABC, m\angle B=90$ and $\overline{\text{BM}}$ is an altitude. If $AB=8$ and $BC=6$, then $BM=\ldots\ldots$

(A) 2.4 (B) 4.8

(C) 6 (D) 7

(28) In $\Delta ABC, m\angle B=90$ and $\overline{BM}$ is an altitude. If $AM=9$ and $CM=16$, find the perimeter of ΔABC.

(A) 60 (B) 65

(C) 70 (D) 75

(29) The midpoint of the line segment joining $(1,1)$ and $(3,3)$ is.........

(A) $(1,1)$ (B) $(2,2)$

(C) $\left(\frac{3}{2},\frac{3}{2}\right)$ (D) $\left(\frac{2}{3},\frac{2}{3}\right)$

(30) Solve the following pairs of equations:
$43x+67y=-24$
$67x+43y=24$

(A) $1,0$ (B) $-1,1$

(C) $1,-1$ (D) $2,-3$

(31) The line segment joining $A(2,2)$ and $B(2,-2)$ intersects ....

(A) $X-$ axis at $(2,0)$ (B) $Y-$ axis at $(0,2)$

(C) $X-$ axis at $(-2,0)$ (D) $Y-$ axis at $(0,-2)$

(32) In $\Delta PQR, m\angle Q : m\angle R : m\angle P = 1:2:1$. If $PQ=2\sqrt{6}$, then $PR=\ldots\ldots\ldots\ldots$

(A) $\sqrt{6}$ (B) $2\sqrt{6}$

(C) $2\sqrt{3}$ (D) $2\sqrt{2}$

(33) P is a point in the exterior of a circle having centre O and radius $21. OP=25$. A tangent from P touches the circle at Q. Find PQ.

(A) 20 (B) 10

(C) 25 (D) 15

(34) How many tangents can a circle have?

(A) $infinite$ (B) 0

(C) 1 (D) 10

(35) The sum of first 20 terms of the $A.P.\ 1, 21, 41, \ldots$ is.......

(A) 3820 (B) 3810

(C) 3835 (D) 3790

(36) $\Delta ABC, A-M-B, A-N-C$ and $\overline{MN}||\overline{BC}$. $AM=6, MB=9$ and $AN=8$, find AC.

(A) 20 (B) 30

(C) 40 (D) 50

(37) In $\Delta ABC, m\angle B=90$. If $AB=4$ and $BC=7.5$ find AC.

(A) 10 (B) 5

(C) 6 (D) 8.5

(38) If the zeros of cubic polynomial $p(x)=ax^3+bx^2+cx+d : a\neq 0$ are α, β and γ. then $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\ldots\ldots\ldots\ldots$

(A) $-\frac{c}{a}$ (B) $-\frac{c}{d}$

(C) $\frac{c}{d}$ (D) $-\frac{b}{d}$

(39) From the top of a hill $100\,m$ high, the angles of depression of the top and the bottom of a tower are observed to be 30 and 45 respectively. Find the height of the tower. (in m)

(A) 30 (B) 56

(C) 42 (D) 38

(40) Solve the following equations using the method of 'completing a square' : $x^2+3x-5=0$.

(A) $\frac{3+\sqrt{29}}{2}$ and $\frac{3+\sqrt{29}}{2}$ (B) $\frac{-5-\sqrt{25}}{2}$ and $\frac{-5+\sqrt{25}}{2}$

(C) $\frac{-3-\sqrt{29}}{2}$ and $\frac{-3+\sqrt{29}}{2}$ (D) $\frac{-3+\sqrt{29}}{2}$ and $\frac{-3+\sqrt{29}}{2}$

(41) The n^{th} term of an $A.P.$ is given by $T_n=3n-1$. For this $A.P.$, the common difference $d=\ldots\ldots\ldots$

(A) -2 (B) 2

(C) -3 (D) 3

(42) The distance between $(7,5)$ and $(2,5)$ is........

(A) 9 (B) 5

(C) 4.5 (D) $\sqrt{13}$

(43) Which constant should be added and subtracted to solve the quadratic equation $4x^2-\sqrt{3}x-5=0$ by the method of completing the square?

(A) $\frac{9}{16}$ (B) $\frac{3}{16}$

(C) $\frac{3}{4}$ (D) $\frac{\sqrt{3}}{4}$

(44) The segment joining $A(-2,1)$ and $B(7,8)$ divided in five congruent segments, then find the coordinates of the third point from A.

(A) $\left(\frac{50}{5},\frac{2}{5}\right)$ (B) $\left(\frac{17}{5},\frac{22}{5}\right)$

(C) $\left(\frac{10}{5},\frac{36}{5}\right)$ (D) $\left(\frac{10}{3},\frac{2}{3}\right)$

(45) Obtain the roots of the following quadratic equations by using the general formula for the solution :
$\sqrt{3}x^2+10x-8\sqrt{3}=0$

(A) $-4\sqrt{3},\frac{2}{\sqrt{3}}$ (B) $-\frac{\sqrt{3}}{2},-2\sqrt{3}$

(C) $-\sqrt{5},\frac{\sqrt{5}}{3}$ (D) $-\sqrt{2},-\frac{5}{\sqrt{2}}$

(46) The ratio of the sums of first n terms of two $A.P.s$ is $\frac{4n+3}{5n-7}$. Find the ratio of 15^{th} terms of the $A.P.s$.

(A) $\frac{229}{140}$ (B) $\frac{190}{130}$

(C) $\frac{119}{138}$ (D) $\frac{220}{138}$

(47) When the angle of elevation of the sun increases from 30 to 60, the shadow of a tower decreases by $50\,m$. Find the height of the tower. (in m)

(A) 43.25 (B) 55.12

(C) 49.23 (D) 39.54

(48) For a given $A.P.$, the common difference is 5 and its 15^{th} term is 72 . Find the first term of the $A.P.$ and its 50^{th} term.

(A) 639 (B) 514

(C) 350 (D) 247

(49) If $\triangle ABC\sim\triangle DEF, AB=4\,cm, DE=6\,cm, EF=9\,cm$ and $FD=12\,cm$ find the perimeter of $\triangle ABC$. (in cm)

(A) 8 (B) 6

(C) 18 (D) 28

(50) Find the 100^{th} term of the $A.P.$ $50,56,62,68,\ldots$

(A) 573 (B) 644

(C) 515 (D) 663

Mathematics - Section B (SUBJECTIVE)

VSQ [1 Mark]

(1) Solve the following pairs of equations:
$x+y=3.3$
$\frac{0.6}{3x-2y}=-1 \quad 3x-2y\neq 0$

(2) Find the area of the triangle whose vertices are $(2,3),(-1,0),(2,-4)$

(3) $a\sin\theta=3$ and $a\cos\theta=4$, then $a=$ .... (where, $a>0$)

(4) If sec $4A=$ cosec $(A-20^\circ)$, where $4A$ is an acute angle, find the value of A. (in $^\circ$)

(5) A ladder is placed leaning on a wall. Its upper end reaches to the height of $12\,m$ on the wall and its lower end rests $9\,m$ away from the base of the wall. Find the...........m length of the ladder.

(6) In ΔABC and ΔPQR, $\angle A\cong\angle P$ and $\angle B\cong\angle R$. If $AB=8, PQ=7.5$ and $AC=6$, find PR

(7) Aruna has only $Rs.\,1$ and $Rs.\,2$ coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs . 75, then the number of $Rs.\,1$ and $Rs.\,2$ coins are, respectively

(8) If $4x-12y=20$, then $5x-15y=\ldots\ldots\ldots\ldots$

(9) If the zeros of the cubic polynomial $p(x)=ax^3+bx^2+cx+d; a\neq 0, a,b,c,d\in R$ are α,β and γ; then $\alpha\beta+\beta\gamma+\gamma\alpha=\ldots\ldots\ldots\ldots$

Mathematics - Section B (SUBJECTIVE)

S.A [2 Marks]

(10) Find two consecutive odd positive integers, sum of whose squares is 290.

(11) The n^{th} term of an $A.P.$ in given by $T_n=3n-1$. Then, the common difference of the $A.P.$ is..........

(12) If $A(-2,-1)$ and $B(7,8)$, then find the coordinates of the trisection points of $\overline{AB}$.

(13) The length of a rectangle is $2\,cm$ less than 3 times its breadth. If its area is $280\,cm^2$. then find its length.

(14) Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

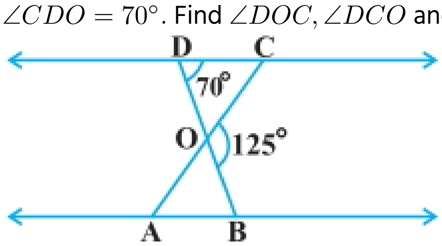
(15) At present Asha's age (in $years$) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha. (in $year$)

(16) Five years ago. the sum of the ages of a father and two sons was x years, then after five years, the sum of the ages of all will be .... years.

(17) Determine if the points $(1,5),(2,3)$ and $(-2,-11)$ are collinear.

(18) Draw the graphs of the pair of linear equations $x+3y=6$ and $2x-3y=12$. Determine the coordinates of the vertices of the triangle formed by these linear equations and the $Y-$axis.

(19) In Figure $\Delta ODC\sim\Delta OBA, \angle BOC=125^\circ$ and $\angle CDO=70^\circ$. Find $\angle DOC, \angle DCO$ and $\angle OAB$



Mathematics - Section B (SUBJECTIVE)

3 marks

(20) Determine, graphically, the vertices of the triangle formed by the lines
$y=x, \quad 3y=x, \quad x+y=8$

(21) Solve the following pairs of equations by reducing them to a pair of linear equations
$\frac{4}{x}+3y=14$
$\frac{3}{x}-4y=23$

(22) If the vertices of $\Delta\,LMN$ are $L(1,4),M(4,1)$ and $N(4,4)$, then $\Delta\,LMN$ is......

Mathematics - Section B (SUBJECTIVE) . 4 marks .

(23) The shadow of a tower standing on a level plane is found to be $50\,m$ longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower. (in m)

(24) The points $A\left(x_1, y_1\right), B\left(x_2, y_2\right)$ and $C\left(x_3 y_3\right)$ are the vertices of $\triangle ABC$.
(i) The median from A meets BC at D. Find the coordinates of the point D.
(ii) Find the coordinates of the point P on AD such that $AP : PD = 2 : 1$
(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that $BQ : QE = 2 : 1$ and $CR : RF = 2 : 1$
(iv) What are the coordinates of the centroid of the triangle ABC ?

(25) From the top of a tower $h\ m$ high, the angles of depression of two objects, which are in line with the foot of the tower are α and $\beta(\beta > \alpha)$. Find the distance between the two objects.

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MCQ and Subjective

(Answer Key)

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Mathematics - Section A (MCQ)

1 - C	2 - C	3 - C	4 - A	5 - B	6 - C	7 - D	8 - C	9 - A	10 - D
11 - A	12 - D	13 - C	14 - A	15 - A	16 - B	17 - C	18 - A	19 - C	20 - A
21 - C	22 - C	23 - C	24 - B	25 - C	26 - C	27 - B	28 - A	29 - B	30 - C
31 - A	32 - C	33 - A	34 - A	35 - A	36 - A	37 - D	38 - B	39 - C	40 - C
41 - D	42 - B	43 - B	44 - B	45 - A	46 - C	47 - A	48 - D	49 - C	50 - B

Global Education of Science

Subject : Mathematics
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MCQ and Subjective

(Solutions)

Paper Set : 1
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.... **Mathematics - Section A** (MCQ) ....

(1) The distance from the origin of $A(a\cos\theta, a\sin\theta)$ is $\ldots\ldots\ldots\ldots (a \in R^+)$

(A) $a\cos\theta$ (B) $a\sin\theta$

(C) a (D) 1

Solution:(Correct Answer:C)

$A(a\cos\theta, a\sin\theta)$ and origin $O(0,0)$

$\therefore$ The required distance $= \sqrt{a^2\cos^2\theta + a^2\sin^2\theta}$

$= \sqrt{a^2\left(\cos^2\theta + \sin^2\theta\right)}$

$= \sqrt{a^2(1)} = a \quad (\because a \in R^+)$

(2) The sum of the two−digit number and the number obtained by interchanging the digits is always divisible by ....

(A) 9 (B) 10

(C) 11 (D) 12

Solution:(Correct Answer:C)

(3) Find the roots of the following quadratic equations by using the general formula for the roots, if they exist :
$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$; $(x \neq -1, -2, -4)$

(A) $-2(1+\sqrt{3})$ and $-2(1+\sqrt{3})$

(B) $2(1-\sqrt{3})$ and $2(1+\sqrt{3})$

(C) $2(1+\sqrt{3})$ and $2(1-\sqrt{3})$

(D) $-2(1-\sqrt{3})$ and $-2(1-\sqrt{3})$

Solution:(Correct Answer:C)

$\frac{1}{x+1} + \frac{2}{x+2} = \frac{4}{x+4}$

$\therefore \frac{x+2+2x+2}{(x+1)(x+2)} = \frac{4}{x+4}$

$\therefore \frac{3x+4}{x^2+3x+2} = \frac{4}{x+4}$

$\therefore (x+4)(3x+4) = 4\left(x^2+3x+2\right)$

$\therefore 3x^2+16x+16 = 4x^2+12x+8$

$\therefore -x^2+4x+8 = 0$

$\therefore x^2-4x-8 = 0$

Comparing $x^2-4x-8=0$ with

$ax^2+bx+c=0; a=1, b=-4, c=-8$

Discriminant $D = b^2-4ac$

$= (-4)^2 - 4(1)(-8)$

$= 16+32$

$= 48 > 0$

since, $D > 0$, the given quadratic equation has distinct real roots.

$\alpha = \frac{-b+\sqrt{D}}{2a}$

$= \frac{-(-4)+\sqrt{48}}{2(1)}$

$= \frac{4+4\sqrt{3}}{2}$

$= \frac{2(2+2\sqrt{3})}{2}$

$= 2+2\sqrt{3}$

$= 2(1+\sqrt{3})$

$\beta = \frac{-b-\sqrt{D}}{2a}$

$= \frac{-(-4)-\sqrt{48}}{2(1)}$

$= \frac{4-4\sqrt{3}}{2}$

$= \frac{2(2-2\sqrt{3})}{2}$

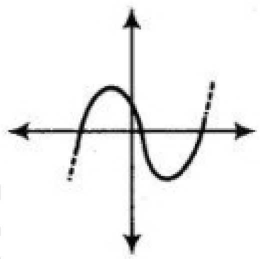
$= 2-2\sqrt{3}$

$= 2(1-\sqrt{3})$

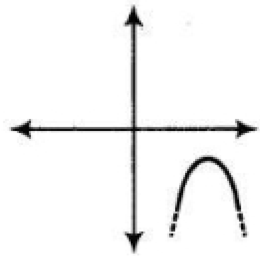
Thus, the roots of the given equation are $2(1+\sqrt{3})$ and $2(1-\sqrt{3})$

(4) Which of the following is not the graph of a quadratic polynomial?

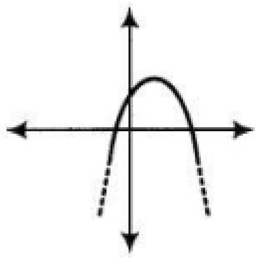
(A)



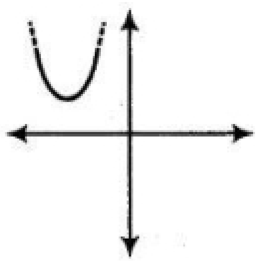
(B)



(C)



(D)



Solution:(Correct Answer:A)

For any quadratic polynomial $ax^2+bx+c, a \neq 0$, the graph of the Corresponding equation $y = ax^2+bx+c$ has one of the two shapes either open upwards like u or open downwards like ∩ depending on whether $a > 0$ or $a < 0$. These curves are called parabolas. So, option (a) cannot be possible.

Also, the curve of a quadratic polynomial crosses the X - axis on at most two points but in option (a) the curve crosses the X - axis on the three points, so it does not represent the quadratic polynomial.

(5) $\Delta ABC \sim \Delta PQR$ for the correspondence $ABC \leftrightarrow PQR$. If $AB=4, BC=8, AC=10$ and $PR=15$, then the perimeter of $\Delta PQR = \ldots\ldots$

(A) 25 **(B)** 33

(C) 40 (D) 60

Solution:(Correct Answer:B)

Perimeter of $\Delta ABC = 4+8+10 = 22$

The correspondence $ABC \leftrightarrow PQR$ is a similarity.

$\therefore \frac{\text{Perimeter of } \Delta ABC}{\text{Perimeter of } \Delta PQR} = \frac{AC}{PR}$.

$\therefore \frac{22}{\text{Perimeter of } \Delta PQR} = \frac{10}{15} \quad \therefore$ Perimeter of $\Delta PQR = 33$

(6) Solve the following pairs of linear equations in two variables using graph : $2x+y=7, \quad x-2y=6$

(A) $(-6,\ 3)$ (B) $(-3,\ 2)$

(C) $(4,\ -1)$ (D) $(7,\ 1)$

Solution:(Correct Answer:C)

$(4,\ -1)$

(7) Which of the following group truely match the data of Part I with the data of Part II?

Part I	Part II
1. In $\Delta ABC, AB = 3, BC = 4$ and $AC = 5$	a. in-radius 2
2. In $\Delta PQR, PQ = 5, QR = 12$ and $PR = 13$	b. in-radius 1
3. In $\Delta XYZ, XY = 8, YZ = 15$ and $XZ = 17$	c. in-radius 6
4. In Δ MNP, $MN = 20, NP = 21$ and $MP = 29$	d. in-radius 3

(A) $(1-a),(2-b),(3-c),(4-d)$

(B) $(1-c),(2-d),(3-a),(4-b)$

(C) $(1-d),(2-c),(3-b),(4-a)$

(D) $(1-b),(2-a),(3-d),(4-c)$

Solution:(Correct Answer:D)

(8) A circle touches the sides $\overline{AB}, \overline{BC}$ and $\overline{CA}$ of ΔABC at the points D, E, F respectively. If $AB=13, BC=12$ and $CA=5$, then $AD = \dots\dots\dots$

(A) 2 (B) 5

(C) 3 (D) 10

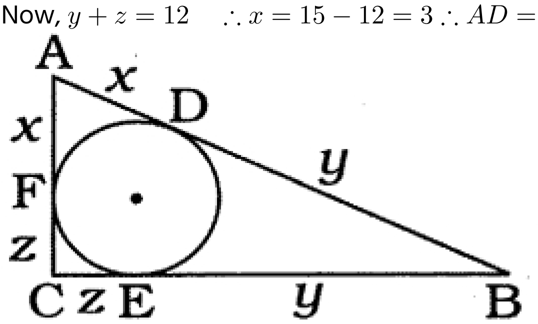
Solution:(Correct Answer:C)

$AB = x+y = 13; \quad BC = y+z = 12$ and $AC = z+x = 5$

$\therefore 2(x+y+z) = 13+12+5 = 30$

$\therefore x+y+z = 15$

Now, $y+z = 12 \quad \therefore x = 15-12 = 3 \therefore AD = 3$



(9) In $\Delta PQR, m\angle Q = 90$ and $\overline{QM}$ is an altitude. If $PM = 8$ and $RM = 12$, then $QM = \dots\dots\dots$

(A) $4\sqrt{6}$ (B) $8\sqrt{3}$

(C) 10 (D) 18

Solution:(Correct Answer:A)

In $\Delta PQR, m\angle Q = 90$ and $\overline{QM}$ is an altitude.

$\therefore QM^2 = PM \cdot RM = 8 \times 12 = 96$

$\therefore QM = \sqrt{96} = \sqrt{16\times 6} = 4\sqrt{6}$

(10) .... is the value of the k, if one of the roots of the quadratic equation $x^2+6x+k=0$ is 4.

(A) 20 (B) 40

(C) 8 **(D)** -40

Solution:(Correct Answer:D)

-40

(11) A ladder is leaning against a wall such that its upper end touches the wall at the height $3m$ and the ladder is inclined at an angle having measure 30 with the ground. Find the length of the ladder. (in m)

(A) 6 (B) 8

(C) 4 (D) 12

Solution:(Correct Answer:A)

Here, $\overline{AC}$ is the ladder and $\overline{AB}$ is the wall.

Then, $AB = 3m, m\angle B = 90$ and $m\angle C = 30$

In $\Delta ABC, m\angle B = 90$

$\therefore \sin C = \frac{AB}{AC}$

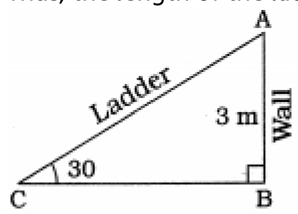
$\therefore \sin \quad 30 = \frac{3}{AC}$

$\therefore \frac{1}{2} = \frac{3}{AC}$

$\therefore AC = 3\times 2$

$\therefore AC = 6$

Thus, the length of the ladder is $6\,m$.



(12) Find the roots of the quadratic equations by using the quadratic formula

$x^2 + 2\sqrt{2}x - 6 = 0$

(A) $\sqrt{7}, -2\sqrt{3}$ (B) $\sqrt{5}, -2\sqrt{3}$

(C) $\sqrt{2}, -2\sqrt{3}$ **(D)** $\sqrt{2}, -3\sqrt{2}$

Solution:(Correct Answer:D)

Given equation is $x^2 + 2\sqrt{2}x - 6 = 0$

On comparing with $ax^2+bx+c=0$, we get

$a=1, b=2\sqrt{2}$ and $c=-6$

By quadratic formula, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \frac{-(2\sqrt{2})\pm\sqrt{(2\sqrt{2})^2-4(1)(-6)}}{2(1)} = \frac{-2\sqrt{2}\pm\sqrt{8+24}}{2}$

$= \frac{-2\sqrt{2}\pm\sqrt{32}}{2} = \frac{-2\sqrt{2}\pm 4\sqrt{2}}{2}$

$= \frac{-2\sqrt{2}+4\sqrt{2}}{2}, \frac{-2\sqrt{2}-4\sqrt{2}}{2} = \sqrt{2}, -3\sqrt{2}$

So, $\sqrt{2}$ and $-3\sqrt{2}$ are the roots of the given equation.

(13) In $\Delta ABC, A-M-B, A-N-C$ and $\overline{MN}\|\overline{BC}$. If $AM = x+3, AB = 2x, AN = x+5$ and $AC = 2x+3$, find the value of x.

(A) 3 (B) 5

(C) 9 (D) 1

Solution:(Correct Answer:C)

(14) The perimeter of rhombus $ABCD$ is 68 . If $AC = 30$, find BD

(A) 16 (B) 30

(C) 35 (D) 45

Solution:(Correct Answer:A)

(15) Solve the following pairs of linear equations in two variables using graph : $x+y=8,\ x-y=2$

(A) $(5,\ 3)$ (B) $(-5,\ 3)$

(C) $(5,\ -3)$ (D) $(-5,\ -3)$

Solution:(Correct Answer:A)

$(5, 3)$

(16) The degree of the polynomial $p(x) = x^2 - x^3 + x + 1$ is ....

(A) -3 **(B)** 3

(C) 2 (D) 1

Solution:(Correct Answer:B)

3

(17) Which of the following equations has two distinct real roots?

(A) $2x^2 - 3\sqrt{2}x + \frac{9}{4} = 0$ (B) $x^2 + 3x + 2\sqrt{2} = 0$

(C) $x^2 + x - 5 = 0$ (D) $5x^2 - 3x + 1 = 0$

Solution:(Correct Answer:C)

(c) The given equation is $x^2 + x - 5 = 0$
On comparing with $ax^2 + bx + c = 0$, we get
$a = 1, b = 1$ and $c = -5$
The discriminant of $x^2 + x - 5 = 0$ is
$D = b^2 - 4ac = (1)^2 - 4(1)(-5)$
$= 1 + 20 = 21$
$\Rightarrow b^2 - 4ac > 0$
So, $x^2 + x - 5 = 0$ has two distinct real roots.
(a) Given equation is, $2x^2 - 3\sqrt{2}x + 9/4 = 0$
On comparing with $ax^2 + bx + c = 0$
$a = 2, b = -3\sqrt{2}$ and $c = 9/4$
Now, $D = b^2 - 4ac = (-3\sqrt{2})^2 - 4(2)(9/4) = 18 - 18 = 0$
Thus, the equation has real and equal roots.
(b) Given equation is $x^2 + 3x + 2\sqrt{2} = 0$
On comparing with $ax^2 + bx + c = 0$
$a = 1, b = 3$ and $c = 2\sqrt{2}$
Now, $D = b^2 - 4ac = (3)^2 - 4(1)(2\sqrt{2}) = 9 - 8\sqrt{2} < 0$
$\therefore$ Roots of the equation are not real.
(d) Given equation is, $5x^2 - 3x + 1 = 0$
On comparing with $ax^2 + bx + c = 0$
$a = 5, b = -3, c = 1$
Now, $D = b^2 - 4ac = (-3)^2 - 4(5)(1) = 9 - 20 < 0$
Hence, roots of the equation are not real.

(18) In $\Delta PQR, \overline{PS}$ is a median. If $PQ = 9, PR = 40$ and $PS = 20.5$, find QR.

(A) 41 (B) 30

(C) 35 (D) 50

Solution:(Correct Answer:A)

(19) Find the sum of last ten terms of the $AP : 8, 10, 12, \ldots, 126$.

(A) 1150 (B) 1160

(C) 1170 (D) 1180

Solution:(Correct Answer:C)

For finding, the sum of last ten terms, we write the given AP in reverse order.
i.e., $126, 124, 122 \ldots 12, 10, 8$
Here, first term $(a) = 126$, common difference,
$(d) = 124 - 126 = -2$
$S_{10} = \frac{10}{2}[2a + (10 - 1)d] \quad [\because S_n = \frac{n}{2}[2a + (n - 1)d]]$
$= 5\{2(126) + 9(-2)\} = 5(252 - 18)$
$= 5 \times 234 = 1170$

(20) The product of the zeros of $p(x) = 4x^2 + 12x + 5$ is.........

(A) $\frac{5}{4}$ (B) $\frac{4}{5}$

(C) $\frac{3}{4}$ (D) $\frac{4}{3}$

Solution:(Correct Answer:A)

(21) Solve the following equations using the method of factorization : $x - \frac{1}{x} = \frac{45}{14} \quad (x \neq 0)$

(A) $-\frac{7}{2}$ and $-\frac{2}{7}$ (B) $\frac{7}{2}$ and $\frac{2}{7}$

(C) $\frac{7}{2}$ and $-\frac{2}{7}$ (D) $-\frac{7}{2}$ and $\frac{2}{7}$

Solution:(Correct Answer:C)

$x - \frac{1}{x} = \frac{45}{14}$
$\therefore \frac{x^2-1}{x} = \frac{45}{14}$
$\therefore 14x^2 - 14 = 45x$
$\therefore 14x^2 - 45x - 14 = 0$
$\therefore 14x^2 - 49x + 4x - 14 = 0$
$\therefore 7x(2x - 7) + 2(2x - 7) = 0$
$\therefore (2x - 7)(7x + 2) = 0$
$\therefore 2x - 7 = 0$ or $7x + 2 = 0$
$\therefore x = \frac{7}{2}$ or $x = -\frac{2}{7}$
The roots of the quadratic equation are $\frac{7}{2}$ and $-\frac{2}{7}$.

(22) g.c.d. $(18, 24) \times 1.c.m.(18, 24) = \ldots\ldots\ldots\ldots$

(A) 144 (B) 72

(C) 432 (D) $6 \times 18 \times 24$

Solution:(Correct Answer:C)

According to formula g.c.d. $(a, b) \times 1.c \cdot m, \quad (a, b) = ab.$
g.c.d. $(18, 24) \times 1.c \cdot m.(18, 24) = 18 \times 24 = 432$

(23) $\Delta XYZ \sim \Delta DEF$ for the correspondence $XYZ \leftrightarrow EDF$. If $XY = 3, YZ = 4, ZX = 6$ and $DF = 12$, find the perimeter of ΔDEF.

(A) 50 (B) 45

(C) 39 (D) 60

Solution:(Correct Answer:C)

39

(24) $(0, 0), (3.1, 0)$ and $(0, 4.5)$ are the vertices of .... triangle.

(A) an equilateral **(B)** a right angled

(C) an isosceles (D) an acute angled

Solution:(Correct Answer:B)

The given points are an origin, on $X-$ axis and on $Y-$ axis.
$\therefore$ The triangle is right angled.

(25) Find the square root in the form of binomial surd : $6 + \sqrt{35}$

(A) $\frac{\sqrt{6}+\sqrt{5}}{2}$ (B) $\frac{\sqrt{4}+\sqrt{10}}{3}$

(C) $\frac{\sqrt{14}+\sqrt{10}}{2}$ (D) $\frac{\sqrt{10}+\sqrt{14}}{2}$

Solution:(Correct Answer:C)

$\frac{\sqrt{14}+\sqrt{10}}{2}$

(26) If 64 is the discriminant of $kx^2 - 4x - 4 = 0$, then the value of k is ....

(A) -4 (B) 4

(C) 3 (D) -3

Solution:(Correct Answer:C)

3

(27) In $\Delta ABC, m\angle B = 90$ and $\overline{\text{BM}}$ is an altitude. If $AB = 8$ and $BC = 6$, then $BM = \ldots\ldots$

(A) 2.4 **(B)** 4.8

(C) 6 (D) 7

Solution:(Correct Answer:B)

In $\Delta ABC, \quad m\angle B = 90$
$\therefore AC^2 = AB^2 + BC^2 = 8^2 + 6^2 = 64 + 36 = 100$
$\therefore AC = 10$
Now, in $\Delta ABC, m\angle B = 90$ and $\overline{BM}$ is an altitude.
$\therefore ABC = \frac{1}{2}AB \times BC = \frac{1}{2}AC \times BM$
$\therefore AB \times BC = AC \times BM \quad \therefore 8 \times 6 = 10 \times BM \quad \therefore BM = 4.8$

(28) In $\Delta ABC, m\angle B = 90$ and $\overline{BM}$ is an altitude. If $AM = 9$ and $CM = 16$, find the perimeter of ΔABC.

(A) 60 (B) 65
(C) 70 (D) 75

Solution:(Correct Answer:A)

(29) The midpoint of the line segment joining $(1,1)$ and $(3,3)$ is.........

(A) $(1,1)$ **(B)** $(2,2)$
(C) $\left(\frac{3}{2}, \frac{3}{2}\right)$ (D) $\left(\frac{2}{3}, \frac{2}{3}\right)$

Solution:(Correct Answer:B)

$(2,2)$

(30) Solve the following pairs of equations:
$43x + 67y = -24$
$67x + 43y = 24$

(A) $1, 0$ (B) $-1, 1$
(C) $1, -1$ (D) $2, -3$

Solution:(Correct Answer:C)

Given pair of linear equations is
$43x + 67y = -24.....(i)$
and $67x + 43y = 24.....(ii)$
On multiplying Eq. (i) by 43 and Eq. (ii) by 67 and then subtracting both of them, we get
$(67)^2x + 43 \times 67y = 24 \times 67$
$(43)^2x + 43 \times 67y = -24 \times 43$
$- \qquad - \qquad +$
____
$\{(67)^2 - (43)^2\} \times = 24(67 + 43)$
$\Rightarrow \quad (67 + 43)(67 - 43)x = 24 \times 110 \quad [\because (a^2 - b^2) = (a - b)(a + b)]$
$\Rightarrow \quad 110 \times 24x = 24 \times 110$
$\Rightarrow \quad x = 1$
Now, put the value of x in Eq. (i), we get
$43 \times 1 + 67y = -24$
$\Rightarrow \quad 67y = -24 - 43$
$\Rightarrow \quad 67y = -67$
$\Rightarrow \quad y = -1$
Hence, the required values of x and y are 1 and -1, respectively.

(31) The line segment joining $A(2,2)$ and $B(2,-2)$ intersects ....

(A) $X-$ axis at $(2,0)$ (B) $Y-$ axis at $(0,2)$
(C) $X-$ axis at $(-2,0)$ (D) $Y-$ axis at $(0,-2)$

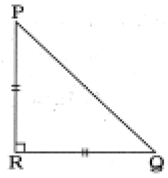
Solution:(Correct Answer:A)

(32) In $\Delta PQR, m\angle Q : m\angle R : m\angle P = 1 : 2 : 1$. If $PQ = 2\sqrt{6}$, then $PR = ...$

(A) $\sqrt{6}$ (B) $2\sqrt{6}$
(C) $2\sqrt{3}$ (D) $2\sqrt{2}$

Solution:(Correct Answer:C)

In $\Delta PQR, \quad m\angle Q : m\angle R : m\angle P = 1 : 2 : 1$
$\therefore m\angle P = m\angle Q = 45$ and $m\angle R = 90$
$\therefore QR = PR$
In $\Delta PQR, \quad m\angle R = 90$
$\therefore PQ^2 = PR^2 + QR^2$
$\therefore (2\sqrt{6})^2 = PR^2 + PR^2 \quad (\because QR = PR)$
$\therefore 2PR^2 = 24 \quad \therefore PR^2 = 12 \quad \therefore PR = 2\sqrt{3}$



(33) P is a point in the exterior of a circle having centre O and radius $21. OP = 25$. A tangent from P touches the circle at Q. Find PQ.

(A) 20 (B) 10
(C) 25 (D) 15

Solution:(Correct Answer:A)

20

(34) How many tangents can a circle have?

(A) $infinite$ (B) 0
(C) 1 (D) 10

Solution:(Correct Answer:A)

A circle can have infinite tangents.

(35) The sum of first 20 terms of the $A.P.\ 1, 21, 41, \ldots$ is.......

(A) 3820 (B) 3810
(C) 3835 (D) 3790

Solution:(Correct Answer:A)

(36) $\Delta ABC, A - M - B, A - N - C$ and $\overline{MN} || \overline{BC}$. $AM = 6, MB = 9$ and $AN = 8$, find AC.

(A) 20 (B) 30
(C) 40 (D) 50

Solution:(Correct Answer:A)

(37) In $\Delta ABC, m\angle B = 90$. If $AB = 4$ and $BC = 7.5$ find AC.

(A) 10 (B) 5
(C) 6 **(D)** 8.5

Solution:(Correct Answer:D)

(38) If the zeros of cubic polynomial $p(x) = ax^3 + bx^2 + cx + d : a \neq 0$ are α, β and γ. then $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = ...$

(A) $-\frac{c}{a}$ **(B)** $-\frac{c}{d}$
(C) $\frac{c}{d}$ (D) $-\frac{b}{d}$

Solution:(Correct Answer:B)

$\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = \frac{c/a}{-d/\alpha} = -\frac{c}{d}$

(39) From the top of a hill $100\,m$ high, the angles of depression of the top and the bottom of a tower are observed to be 30 and 45 respectively. Find the height of the tower. (in m)

(A) 30 (B) 56
(C) 42 (D) 38

Solution:(Correct Answer:C)

Here, $\overline{AC}$ is the hill and $\overline{ED}$ is the tower.

$\overline{EB} \perp \overline{AC}, B \in \overline{AC}$

Then, $AC = 100m$

$m\angle B = m\angle C = 90$

$m\angle XAE = 30$ and

$m\angle XAD = 45$

$\therefore m\angle AEB = m\angle XAE = 30$ and

$m\angle ADC = m\angle XAD = 45$ (alternate angles)

In $\Delta ACD, \quad m\angle C = 90$

$\therefore \cot D = \frac{DC}{AC}$

$\therefore \cot 45 = \frac{DC}{100}$

$\therefore 1 = \frac{DC}{100}$

$\therefore DC = 100m$

Then, $\quad EB = DC = 100m$

In $\Delta ABE, m\angle B = 90$

$\therefore \tan E = \frac{AB}{EB}$

$\therefore \tan 30 = \frac{AB}{100}$

$\therefore \frac{1}{\sqrt{3}} = \frac{AB}{100}$

$\therefore AB = \frac{100}{\sqrt{3}}$

$\therefore AB = 100 \times 0.58$

$\therefore AB = 58m$

Now, height of the tower $= ED$

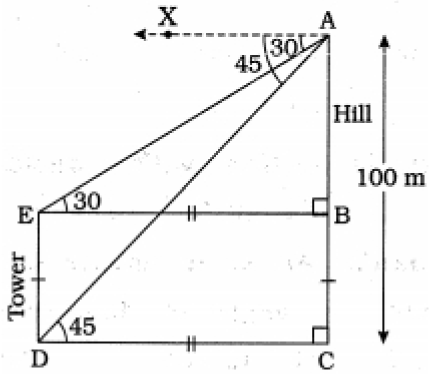
$= BC$

$= AC - AB$

$= 100 - 58$

$= 42m$

Thus, the height of the tower is $42\,m$.



(40) Solve the following equations using the method of 'completing a square' : $x^2 + 3x - 5 = 0$.

(A) $\frac{3+\sqrt{29}}{2}$ and $\frac{3+\sqrt{29}}{2}$ (B) $\frac{-5-\sqrt{25}}{2}$ and $\frac{-5+\sqrt{25}}{2}$

(C) $\frac{-3-\sqrt{29}}{2}$ and $\frac{-3+\sqrt{29}}{2}$ (D) $\frac{-3+\sqrt{29}}{2}$ and $\frac{-3+\sqrt{29}}{2}$

Solution:(Correct Answer:C)

$x^2 + 3x - 5 = 0$

$\therefore x^2 + 3x + \frac{9}{4} - 5 - \frac{9}{4} = 0$

($\because$ Third term $= \frac{(M.T.)^2}{4\times F.T.} = \frac{(3\times)^2}{4x^2} = \frac{9}{4}$)

$\therefore \left(x + \frac{3}{2}\right)^2 - \frac{29}{4} = 0$

$\therefore \left(x + \frac{3}{2}\right)^2 - \left(\frac{\sqrt{29}}{2}\right)^2 = 0$

$\therefore \left(x + \frac{3}{2} + \frac{\sqrt{29}}{2}\right)\left(x + \frac{3}{2} - \frac{\sqrt{29}}{2}\right) = 0$

$\therefore \left(x + \frac{3+\sqrt{29}}{2}\right)\left(x - \frac{-3+\sqrt{29}}{2}\right) = 0$

$\therefore x = \frac{-3-\sqrt{29}}{2}$ or $x = \frac{-3+\sqrt{29}}{2}$

Thus, the solutions of the given equation are $\frac{-3-\sqrt{29}}{2}$ and $\frac{-3+\sqrt{29}}{2}$.

(41) The n^{th} term of an $A.P.$ is given by $T_n = 3n - 1$. For this $A.P.$, the common difference $d = \ldots\ldots\ldots$

(A) -2 (B) 2

(C) -3 **(D)** 3

Solution:(Correct Answer:D)

(42) The distance between $(7, 5)$ and $(2, 5)$ is........

(A) 9 **(B)** 5

(C) 4.5 (D) $\sqrt{13}$

Solution:(Correct Answer:B)

(43) Which constant should be added and subtracted to solve the quadratic equation $4x^2 - \sqrt{3}x - 5 = 0$ by the method of completing the square?

(A) $\frac{9}{16}$ **(B)** $\frac{3}{16}$

(C) $\frac{3}{4}$ (D) $\frac{\sqrt{3}}{4}$

Solution:(Correct Answer:B)

(44) The segment joining $A(-2, 1)$ and $B(7, 8)$ divided in five congruent segments, then find the coordinates of the third point from A.

(A) $\left(\frac{50}{5}, \frac{2}{5}\right)$ **(B)** $\left(\frac{17}{5}, \frac{22}{5}\right)$

(C) $\left(\frac{10}{5}, \frac{36}{5}\right)$ (D) $\left(\frac{10}{3}, \frac{2}{3}\right)$

Solution:(Correct Answer:B)

$\left(\frac{17}{5}, \frac{22}{5}\right)$

(45) Obtain the roots of the following quadratic equations by using the general formula for the solution :

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

(A) $-4\sqrt{3}, \frac{2}{\sqrt{3}}$ (B) $-\frac{\sqrt{3}}{2}, -2\sqrt{3}$

(C) $-\sqrt{5}, \frac{\sqrt{5}}{3}$ (D) $-\sqrt{2}, -\frac{5}{\sqrt{2}}$

Solution:(Correct Answer:A)

$-4\sqrt{3}, \frac{2}{\sqrt{3}}$

(46) The ratio of the sums of first n terms of two $A.P.s$ is $\frac{4n+3}{5n-7}$. Find the ratio of 15^{th} terms of the $A.P.s$.

(A) $\frac{229}{140}$ (B) $\frac{190}{130}$

(C) $\frac{119}{138}$ (D) $\frac{220}{138}$

Solution:(Correct Answer:C)

Suppose, the two A.P.s are $a, a+d, a+2d, \ldots$

and $A, A+D, A+2D, \ldots$

For the first A.P., $T_{15} = a + 14d$ as

$T_n = a + (n-1)d$

For the second A.P., $T_{15} = A + 14D$, as

$T_n = a + (n-1)d$

Now, $S_n = \frac{1}{2}n[2a + (n-1)d]$

$\therefore S_n = \frac{1}{2}n[2a + (n-1)d]$ for the first A.P. and

$S_n = \frac{1}{2}n[2A + (n-1)D]$ for the second A.P.

By the given information,

$\frac{\frac{1}{2}n[2a+(n-1)d^1}{\frac{1}{2}n[2A+(n-1)D]} = \frac{4n+3}{5n-7}$

$\therefore \frac{2a+(n-1)d}{2A+(n-1)D} = \frac{4n+3}{5n-7}$

Taking, $n = 29(2 \times 15 - 1)$,

$\frac{2a+(29-1)d}{2A+(29-1)D} = \frac{4(29)+3}{5(29)-7}$

$\therefore \frac{2a+28d}{2A+28D} = \frac{116+3}{145-7}$

$\therefore \frac{a+14d}{A+14D} = \frac{119}{138}$

Thus, the ratio of the 15^{th} terms of the given $A.P.s$ is $\frac{119}{138}$.

(47) When the angle of elevation of the sun increases from 30 to 60, the shadow of a tower decreases by $50\,m$. Find the height of the tower. (in m)

(A) 43.25 (B) 55.12

(C) 49.23 (D) 39.54

Solution:(Correct Answer:A)

$43.25m$

(48) For a given $A.P.$, the common difference is 5 and its 15^{th} term is 72 . Find the first term of the $A.P.$ and its 50^{th} term.

(A) 639 (B) 514

(C) 350 **(D)** 247

Solution:(Correct Answer:D)

For the given $A.P.$, the common difference $d = 5$ and the 15^{th} term $T_{15} = 72$

Now, $T_n = a + (n-1)d$

$\therefore T_{15} = a + 14d$

$\therefore 72 = a + 14(5)$

$\therefore 72 = a + 70$

$\therefore a = 2$

Now, $T_{50} = a + 49d$

$= 2 + 49(5)$

$= 2 + 245$

$= 247$

Thus, the first term of the given $A.P.$ is 2 and its 50^{th} term is 247.

(49) If $\triangle ABC \sim \triangle DEF, AB = 4\,cm, DE = 6\,cm, EF = 9\,cm$ and $FD = 12\,cm$ find the perimeter of $\triangle ABC$. (in cm)

(A) 8 (B) 6

(C) 18 (D) 28

Solution:(Correct Answer:C)

Given $AB = 4\,cm, DE = 6\,cm$ and $EF = 9\,cm$ and $FD = 12\,cm$ Also, $\triangle ABC \sim \triangle DEF$

$\triangle ABC \sim \triangle DEF$

$\frac{AB}{ED} = \frac{BC}{EF} = \frac{AC}{DF}$

$\frac{4}{6} = \frac{BC}{9} = \frac{AC}{12}$

On taking first two terms, we get

$\frac{4}{6} = \frac{BC}{9}$

$BC = \frac{4\times 9}{6} = 6\,cm$

$= AC = \frac{6\times 12}{9} = 8\,cm$

Now, perimeter of $\triangle ABC = AB + BC + AC$

$= 4 + 6 + 8 = 18\,cm$

(50) Find the 100^{th} term of the $A.P.$ $50, 56, 62, 68, \ldots$

(A) 573 **(B)** 644

(C) 515 (D) 663

Solution:(Correct Answer:B)

644

Mathematics - Section B (SUBJECTIVE)

VSQ [1 Mark]

(1) Solve the following pairs of equations:

$x + y = 3.3$

$\frac{0.6}{3x - 2y} = -1 \quad 3x - 2y \neq 0$

Solution:

Given pair of linear equations are is

$x + y = 33$

and $\frac{0.6}{3x-2y} = -1$

$0.6 = -3x + 2y$

$3x - 2y = -0.6$

$5x = 6 \Rightarrow x = \frac{6}{5} = 1.2$

Now, put the value of x in Eq. (i), we get

$1.2 + y = 3.3$

$\Rightarrow y = 3.3 - 12$

$\Rightarrow y = 2.1$

Hence, the required values of x and y are 1.2 and 2.1, respectively.

(2) Find the area of the triangle whose vertices are $(2,3), (-1,0), (2,-4)$

Solution:

Area of a triangle is given by

Area of a triangle

$= \frac{1}{2}\{x_1(y_2 - y_1) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\}$

Area of the given triangle

$= \frac{1}{2}[2\{0 - (-4)\} + (-1)\{(-4) - (3)\} + 2(3 - 0)]$

$= \frac{1}{2}\{8 + 7 + 6\}$

$= \frac{1}{2}\{8 + 7 + 6\}$

$= \frac{21}{2}$ square units

(3) $a \sin\theta = 3$ and $a \cos\theta = 4$, then $a = \ldots\ldots\ldots\ldots$ (where, $a > 0$)

Solution:

$a \sin\theta = 3 \therefore a^2 \sin^2\theta = 9$

$a \cos\theta = 4 \therefore a^2 \cos^2\theta = 16$

$\therefore a^2 \sin^2\theta + a^2\cos^2\theta = 9 + 16$

$\therefore a^2\left(\sin^2\theta + \cos^2\theta\right) = 25$

$\therefore a^2(1) = 25$

$\therefore a = 5$

(4) If $\sec 4A = \operatorname{cosec}(A - 20^\circ)$, where $4A$ is an acute angle, find the value of A. (in $^\circ$)

Solution:

Given that,

$\sec 4A = \operatorname{cosec}(A - 20^\circ)$

$\operatorname{cosec}(90^\circ - 4A) = \operatorname{cosec}(A - 20^\circ)$

$90^\circ - 4A = A - 20^\circ$

$110^\circ = 5A$

$A = 22^\circ$

(5) A ladder is placed leaning on a wall. Its upper end reaches to the height of $12\,m$ on the wall and its lower end rests $9\,m$ away from the base of the wall. Find the..........m length of the ladder.

Solution:

Here, $\overline{AB}$ represents the part of the wall, $\overline{AC}$ represents the ladder and C is the lower end of the ladder.

$\therefore AB = 12m, \quad BC = 9m$ and $m\angle B = 90$

In $\Delta ABC, \quad m\angle B = 90$

$\therefore AC^2 = AB^2 + BC^2$

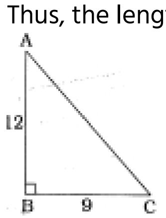
$= 12^2 + 9^2$

$= 144 + 81 = 225$

$\therefore AC = \sqrt{225}$

$\therefore AC = 15$

Thus, the length of the ladder is $15\,m$.



(6) In ΔABC and $\Delta PQR, \angle A \cong \angle P$ and $\angle B \cong \angle R$. If $AB = 8, PQ = 7.5$ and $AC = 6$, find PR

Solution:

In ΔABC and $\Delta PQR, \angle A \cong \angle P$ and $\angle B \cong \angle R$
By AA corollary, the correspondence $ABC \leftrightarrow PRQ$ between ΔABC and ΔPQR is a similarity.
$\therefore \frac{AB}{PR} = \frac{AC}{PQ}$
$\therefore \frac{8}{PR} = \frac{6}{7.5}$
$\therefore 8 \times \frac{7.5}{6} = PR$
$\therefore PR = 10$

(7) Aruna has only $Rs.1$ and $Rs.2$ coins with her. If the total number of coins that she has is 50 and the amount of money with her is Rs . 75, then the number of $Rs.1$ and $Rs.2$ coins are, respectively

Solution:

Let number of $Rs.1$ coins $= x$
and number of $Rs.2$ coins $= y$
Now, by given conditions $x + y = 50$
Also, $x \times 1 + y \times 2 = 75$
$\Rightarrow x + 2y = 75$
On subtracting Eq. (i) from Eq. (ii), we get
$(x + 2y) - (x + y) = 75 - 50$
$\Rightarrow \quad y = 25$
When $y = 25$, then $x = 25$

(8) If $4x - 12y = 20$, then $5x - 15y = \ldots\ldots\ldots\ldots$

(9) If the zeros of the cubic polynomial $p(x) = ax^3 + bx^2 + cx + d; a \neq 0, a, b, c, d \in R$ are α, β and γ; then $\alpha\beta + \beta\gamma + \gamma\alpha = \ldots\ldots\ldots\ldots$

Mathematics - Section B (SUBJECTIVE)

S.A [2 Marks]

(10) Find two consecutive odd positive integers, sum of whose squares is 290.

Solution:

Let the smaller of the two consecutive odd positive integers be x. Then, the second integer will be $x + 2$. According to the question,
$x^2 + (x + 2)^2 = 290$
i.e.,$x^2 + x^2 + 4x + 4 = 290$
i.e.,$2x^2 + 4x - 286 = 0$
i.e.,$x^2 + 2x - 143 = 0$
which is a quadratic equation in x.
Using the quadratic formula, we get
$x = \frac{-2 \pm \sqrt{4+572}}{2} = \frac{-2 \pm \sqrt{576}}{2} = \frac{-2 \pm 24}{2}$
i.e.,$x = 11$ or $x = -13$
But x is given to be an odd positive integer. Therefore, $x \neq -13, x = 11$
Thus, the two consecutive odd integers are 11 and 13.
Check : $11^2 + 13^2 = 121 + 169 = 290$.

(11) The n^{th} term of an $A.P.$ in given by $T_n = 3n - 1$. Then, the common difference of the $A.P.$ is..........

Solution:

$T_n = 3n - 1$
$\therefore T_2 = 3(2) - 1 = 5$ and $T_1 = 3(1) - 1 = 2$
Then, $\quad d = T_2 - T_1 = 5 - 2 = 3$

(12) If $A(-2, -1)$ and $B(7, 8)$, then find the coordinates of the trisection points of $\overline{AB}$.

Solution:

$(1, 2), (4, 5)$

(13) The length of a rectangle is $2\,cm$ less than 3 times its breadth. If its area is $280\,cm^2$. then find its length.

Solution:

Let the breadth of the rectangle be $x\,cm$.
Then, by the given condition, the length of the rectangle is $(3x - 2)\,cm$.
Now, by the data, the area of the rectangle is $280\,cm^2$.
$\therefore$ Area of rectangle $= 280$
$\therefore$ Length $\times$ Breadth $= 280$
$\therefore (3x - 2)(x) = 280$
$\therefore 3x^2 - 2x = 280$
$\therefore 3x^2 - 2x - 280 = 0$
$\therefore (3x + 28)(x - 10) = 0$
$\therefore 3x + 28 = 0$ or $x - 10 = 0$
$\therefore 3x = -28$ or $x = 10$
$\therefore x = -\frac{28}{3}$ or $x = 10$
As x is the breadth of a rectangle, it cannot be negative.
$\therefore x = -\frac{28}{3}$ is not possible.
$\therefore x = 10$
Now, the length of the rectangle
$= 3x - 2$
$= 3(10) - 2 = 28$
Hence, the length of the given rectangle is $28\,cm$.

(14) Determine the AP whose third term is 16 and the 7^{th} term exceeds the 5^{th} term by 12.

Solution:

$= a_3 = 16$
$a + (3 - 1)d = 16$
$a + 2d = 16(1)$
$a_7 - a_5 = 12$
$[a + (7 - 1)d] - [a + (5 - 1)d] = 12$
$(a + 6d) - (a + 4d) = 12$
$2d = 12$
$d = 6$
From equation (1), we obtain
$a + 2(6) = 16$
$a + 12 = 16$
$a = 4$
Therefore,$A.P.$ will be
$4, 10, 16, 22, \ldots$

(15) At present Asha's age (in $years$) is 2 more than the square of her daughter Nisha's age. When Nisha grows to her mother's present age, Asha's age would be one year less than 10 times the present age of Nisha. Find the present ages of both Asha and Nisha. (in $year$)

Solution:

Let Nisha's present age be x yr.
Then, Asha's present age $= x^2 + 2$ [by given condition]
Now, when Nisha grows to her mother's present age i.e., after $\left[(x^2 + 2) - x\right]\ yr$. Then, Asha's age also increased by $\left[(x^2 + 2) - x\right]\ yr$
Again by given condition,
Age of Asha $=$ One years less than 10 times the present age of Nisha
$(x^2 + 2) + \left\{(x^2 + 2) - x\right\} = 10x - 1$
$\Rightarrow 2x^2 - x + 4 = 10x - 1$
$\Rightarrow 2x^2 - 11x + 5 = 0$
$\Rightarrow 2x^2 - 10x - x + 5 = 0$
$\Rightarrow 2x(x - 5) - 1(x - 5) = 0$
$\Rightarrow (x - 5)(2x - 1) = 0$
$\therefore x = 5$

[nere, $x=\frac{1}{2}$ cannot be possible, because at $x=\frac{1}{2}$, Asha's age is $2\frac{1}{4}\ yr$ which is not possible]
Hence, required age of Nisha $=5\,yr$
and required age of Asha $=x^2+2=(5)^2+2=25+2=27\ yr$

(16) Five years ago. the sum of the ages of a father and two sons was x years, then after five years, the sum of the ages of all will be .... years.

(17) Determine if the points $(1,5),(2,3)$ and $(-2,-11)$ are collinear.

Solution:

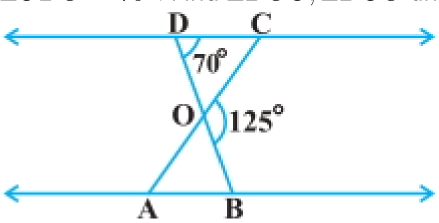
Let the points $(1,5),(2,3)$, and $(-2,-11)$ be representing the vertices A,B, and C of the given triangle respectively.
Let $A=(1,5),B=(2,3),C=(-2,-11)$
$\therefore AB=\sqrt{(1-2)^2+(5-3)^2}=\sqrt{5}$
$BC=\sqrt{(2-(-2))^2+(3-(-11))^2}=\sqrt{4^2+14^2}=\sqrt{16+196}=\sqrt{212}$
$CA=\sqrt{(1-(-2))^2+(5-(-11))^2}=\sqrt{3^2+16^2}=\sqrt{9+256}=\sqrt{265}$
since $AB+BC\neq CA$.
Therefore, the points $(1,5),(2,3)$, and $(-2,-11)$ are not collinear.

(18) Draw the graphs of the pair of linear equations $x+3y=6$ and $2x-3y=12$. Determine the coordinates of the vertices of the triangle formed by these linear equations and the $Y-$ axis.

Solution:

$(6,0),(0,-4),(0,2)$

(19) In Figure $\Delta ODC\sim\Delta OBA,\angle BOC=125^\circ$ and $\angle CDO=70^\circ$. Find $\angle DOC,\angle DCO$ and $\angle OAB$



Solution:

$\angle DOC+125^\circ=180^\circ$ (linear pair)
$\Rightarrow\angle DOC=180^\circ-125^\circ=55^\circ$
In $\triangle DOC$
$\angle DCO+\angle CDO+\angle DOC=180^\circ$ (sum of three angles of $\triangle ODC$)
$\Rightarrow\angle DCO+70^\circ+55^\circ=180^\circ$
$\Rightarrow\angle DCO+125^\circ=180^\circ$
$\Rightarrow\angle DCO=180^\circ-125^\circ=55^\circ$
Now we are given that $\triangle ODC\sim\triangle OBA$
$\Rightarrow\angle OCD=\angle OAB$ (Corresponding angles of similar triangles)
$\Rightarrow\angle OAB=\angle OCD=\angle DCO=55^\circ$
i.e., $\angle OAB=55^\circ$
Hence we have,
$\angle DOC=55^\circ;\angle DCO=55^\circ;\angle OAB=55^\circ$

Mathematics - Section B (SUBJECTIVE) 3 marks

(20) Determine, graphically, the vertices of the triangle formed by the lines
$y=x,\quad 3y=x,\quad x+y=8$

Solution:

Given linear equations are
$y=x...(i)$
$3y=x...(ii)$
and $x+y=8...(iii)$
For equation $y=x$
If $x=1$, then $y=1$
If $x=0$, then $y=0$
If $x=2$, then $y=2$
Table for line $y=x$

x	0	1	2
y	0	1	2
Points	0	A	B

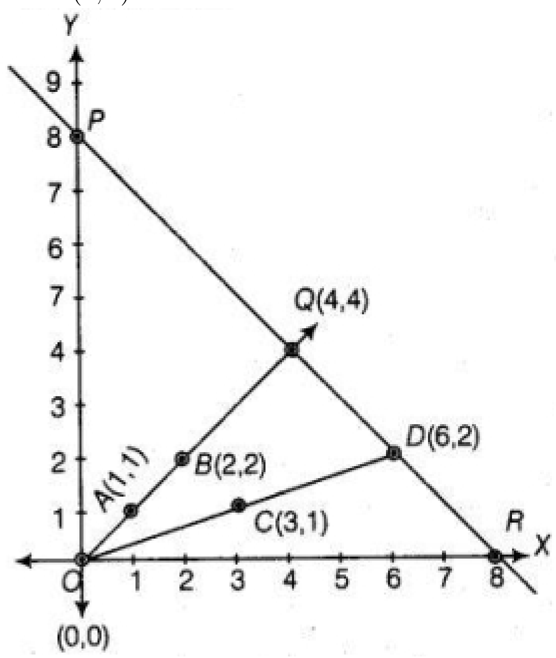
For equation $x=3y$,
If $x=0$, then $y=0$; if $x=3$, then $y=1$ and if $x=6$, then $y=2$
Table for line $x=3y$,

x	0	3	6
y	0	1	2
Points	0	C	D

For equation. $x+y=8\Rightarrow y=8-x$
If $x=0$, then $y=8$, if $x=8$, then $y=0$ and if $x=4$, then $y=4$
Table for line $x+y=8$
For equation, $x+y=8\Rightarrow y=8-x$
If $x=0$, then $y=8$; if $x=8$, then $y=0$ and if $x=4$, then $y=4$
Table for line $x+y=8$.

x	0	4	8
y	8	4	0
Points	P	Q	R

Plotting the points $A(1,1)$ and $6(2,2)$, we get the straight line AB. Plotting the points $C(3,1)$ and $0(6,2)$, we get the straight line CD. Plotting the points $P(0,8),Q(4,4)$ and $R\{8,0\,)$, we
get the straight line PQR. We see that lines AB and CD intersecting the line PR on Q and D, respectively.
So, $AOQD$ is formed by these lines. Hence, the vertices of the $AOOD$ formed by the given lines are $O(0,0),Q(4,4)$ and $D(6,2)$



(21) Solve the following pairs of equations by reducing them to a pair of linear equations
$\frac{4}{x}+3y=14$
$\frac{3}{x}-4y=23$

Solution:

$\frac{4}{x}+3y=14$

$\frac{3}{x}-4y=23$

Substituting $\frac{1}{x}=p$ in the given equations, we obtain

$4p+3y=14 \quad \Rightarrow \quad 4p+3y-14=0 \ldots(1)$

$3p-4y=23 \quad \Rightarrow \quad 3p-4y-23=0 \ldots(2)$

By cross-multiplication, we obtain

$\frac{p}{-69-56}=\frac{y}{-42-(-92)}=\frac{1}{-16-9}$

$\frac{p}{-125}=\frac{y}{50}=\frac{-1}{25}$

$\frac{p}{-125}=\frac{-1}{25}$ and $\frac{y}{50}=\frac{-1}{25}$

$p=5$ and $y=-2$

$p=\frac{1}{x}=5$

$x=\frac{1}{5}$

$y=-2$

(22) If the vertices of $\Delta\, LMN$ are $L(1,4), M(4,1)$ and $N(4,4)$, then $\Delta\, LMN$ is.......

Mathematics - Section B (SUBJECTIVE) . 4 marks .

(23) The shadow of a tower standing on a level plane is found to be $50\,m$ longer when Sun's elevation is 30° than when it is 60°. Find the height of the tower. (in m)

Solution:

Let the height of the tower be h and $RQ=x\,m$

Given that, $PR=50\,m$

and $\angle SPQ=30^{\circ}, \angle SAQ=60^{\circ}$

Now, in $\triangle SRQ$. tan $60^{\circ}=\frac{SQ}{RQ}$

$\Rightarrow \sqrt{3}=\frac{h}{x} \Rightarrow x=\frac{h}{\sqrt{3}}$ ....(i)

and in $\triangle SPQ$, tan $30^{\circ}=\frac{SQ}{PQ}=\frac{SQ}{PR+RQ}=\frac{h}{50+x}$

$\frac{1}{\sqrt{3}}=\frac{h}{50+x}$

$\sqrt{3}\cdot h=50+x$

$\sqrt{3}\cdot h=50+\frac{h}{\sqrt{3}}$ [from Eq.(i)]

$\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)h=50$

$\frac{(3-1)}{\sqrt{3}}h=50$

$h=\frac{50\sqrt{3}}{2}$

$h=25\sqrt{3}\,m$

Hence, the required height of tower is $25\sqrt{3}m$.

(24) The points $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3 y_3)$ are the vertices of $\triangle ABC$.

(i) The median from A meets BC at D. Find the coordinates of the point D.

(ii) Find the coordinates of the point P on AD such that $AP:PD=2:1$

(iii) Find the coordinates of points Q and R on medians BE and CF, respectively such that $BQ:QE=2:1$ and $CR:RF=2:1$

(iv) What are the coordinates of the centroid of the triangle ABC ?

Solution:

Given that, the points $A(x_1,y_1), B(x_2,y_2)$ and $C(x_3,y_3)$ are the vertices of $\triangle ABC$.

(i) We know that, the median bisect the line segment into two equal parts i.e., here D is the mid-point of BC.

$\therefore$ Coordinate of mid-point of $BC=\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$

$\Rightarrow \quad D=\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$

(ii) Let the coordinates of a point P be (x,y)

Given that, the point $P(x,y)$, divide the line joining $A(x_1,y_1)$ and $D\left(\frac{x_2+x_3}{2},\frac{y_2+y_3}{2}\right)$ in the ratio $2:1$, then the coordinates of P

$=\left[\frac{2\cdot\left(\frac{x_2+x_3}{2}\right)+1\cdot x_1}{2+1},\frac{2\cdot\left(\frac{y_2+y_3}{2}\right)+1\cdot y_1}{2+1}\right]$

[$\because$ internal section formula $=\left(\frac{m_1x_2+m_2x_1}{m_1+m_2},\frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$]

$=\left(\frac{x_2+x_3+x_1}{3},\frac{y_2+y_3+y_1}{2}\right)$

$\therefore$ So, required coordinates of point

$P=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

(iii) Let the coordinates of a point Q be (p,q)

Given that, the point $Q(p,q)$, divide the line joining $B(x_2,y_2)$ and $E\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$ in the ratio $2:1$, then the coordinates of Q

$=\left[\frac{2\cdot\left(\frac{x_1+x_3}{2}\right)+1\cdot x_2}{2+1},\frac{2\cdot\left(\frac{y_1+y_2}{2}\right)+1\cdot y_2}{2+1}\right]$

$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

[since, BE is the median of side CA, so BE divides AC in to two equal parts.

$\therefore$ mid-point of $AC=$

Coordinate of $E \Rightarrow E=\left(\frac{x_1+x_3}{2},\frac{y_1+y_3}{2}\right)$]

So, the required coordinate of point

$Q=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

Now, let the coordinates of a point E be (α,β). Given that, the point $R(\alpha,\beta)$, divide the line joining $C(x_3,y_3)$ and $F\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$ in the ratio $2:1$, then the coordinates of R

$=\left[\frac{2\cdot\left(\frac{x_1+x_2}{2}\right)+1\cdot x_3}{2+1},\frac{2\cdot\left(\frac{y_1+y_2}{2}\right)+1\cdot y_3}{2+1}\right]$

$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

[since, CF is the median of side AB. So, CF divides AB in to two equai parts. $\therefore$ mid-point of $AB=$ coordinate of $F \Rightarrow F=\left(\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right)$]

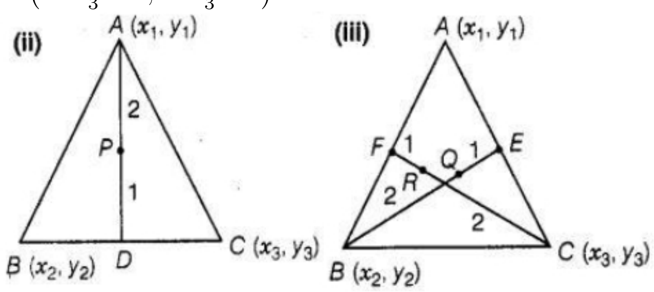
So, the required coordinate of point

$R=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$

(iv) Coordinate of the centroid of the $\triangle ABC$

$=\left(\frac{\text{Sum of abscissa of all vertices, Sum of ordinate of all vertices}}{3}\right)$

$=\left(\frac{x_1+x_2+x_3}{3},\frac{y_1+y_2+y_3}{3}\right)$



(25) From the top of a tower $h\,m$ high, the angles of depression of two objects, which are in line with the foot of the tower are α and $\beta(\beta>\alpha)$. Find the distance between the two objects.

Solution:

Let the distance between two objects is $x\,m$,

and $CD=y\,m$

Given that, $\angle BAX=\alpha=\angle ABD$, [alternate angle]

$\angle CAY=p=\angle ACD$ [alternate angle]

Now, in $\triangle ACD$,

$\tan\beta=\frac{AD}{CD}=\frac{h}{y}$

$y=\frac{h}{\tan\beta}$ ....(i)

and in $\triangle ABD$,

$\tan\alpha=\frac{AD}{BD} \quad \Rightarrow= \frac{AD}{BC+CD}$

$\tan\alpha=\frac{h}{x+y} \Rightarrow x+y=\frac{h}{\tan\alpha}$

$y=\frac{h}{\tan\alpha}-x$ .....(ii)

From Eqs. (i) and (ii),

$\frac{h}{\tan\beta}=\frac{h}{\tan\alpha}-x$

$x=\frac{h}{\tan\alpha}-\frac{h}{\tan\beta}$

$=h\left(\frac{1}{\tan\alpha}-\frac{1}{\tan\beta}\right)=h(\cot\alpha-\cot\beta) \quad [\because \cot\theta=\frac{1}{\tan\theta}]$

which is the required distance between the two objects.
Hence proved.